DOI: 10.5862/JPM.253.3
UDC 536.24

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HYDROMAGNETIC AXISYMMETRIC SLIP FLOW ALONG A VERTICAL STRETCHING CYLINDER WITH A CONVECTIVE BOUNDARY CONDITION

Steady axisymmetric laminar boundary-layer slip flow of a viscous incompressible fluid and heat transfer towards a vertical stretching cylinder in the presence of a uniform magnetic field is investigated. It is assumed that the left surface of the cylinder is heated by a hot convective flow. Using a similarity transformation, the governing system of partial differential equations is first transformed into a system of coupled nonlinear ordinary differential equations. The resulting intricate nonlinear boundary value problem is solved numerically by the fourth-order Runge – Kutta method with the shooting iteration technique. The analytical solutions are presented for a special case. The effects of various physical parameters on the velocity and temperature profiles are discussed through graphs. The values of the skin friction coefficient and the Nusselt number are tabulated and examined. It is found that the thermal boundary layer thickness increases with an increase in the velocity slip, the magnetic field, the surface convection parameter and the curvature parameter and decreases with the Prandtl number.

AXISYMMETRIC FLOW, MHD, SLIP FLOW, STRETCHING CYLINDER, CONVECTIVE BOUNDARY.

1. Introduction

Flow and heat transfer due to a stretching cylinder or a flat plate in a quiescent or moving fluid is important in a number of industrial manufacturing processes that include manufacturing both metal and polymer sheets. Flow over a cylinder is considered to be two-dimensional if the body's radius is large and comparable to the boundary-layer thickness. On the other hand, for a thin or a slender cylinder, the radius of the cylinder can be of the same order as the boundary-layer thickness. Therefore, the flow may be considered axisymmetric instead of two-dimensional along a vertical or a horizontal cylinder [1 – 6].

Magnetohydrodynamic (MHD) flow and heat transfer for fluid have broad applications in various engineering problems such as MHD power generators, petroleum industries, plasma studies, geothermal energy extractions, boundary layer control in the field of aerodynamics, and many others [7–27]. MHD flow along a stretching cylinder has been considered by the authors of Refs. [28–31].

The nonadherence of the fluid to a solid boundary, known as velocity slip, is a phenomenon that has been observed under certain circumstances. When fluid is encountered in microelectromechanical systems, the no-slip condition at the solid-fluid interface is abandoned in favor of a slip flow model which represents the nonequilibrium region near the interface more accurately. In all of the above-mentioned investigations, the no-slip condition at the boundary was assumed. Even in literature, the slip flow over a flat plate has not been studied sufficiently. Zheng, et al. [32] investigated the flow and the radiation heat transfer of nanofluid over a stretching sheet with the velocity slip and the temperature jump in a porous medium. Mukhopadhyay [33] analyzed the slip effects on MHD flow along a horizontal stretching cylinder. Very recently, Abbas, et al. [34] studied the slip effects on the flow over an unsteady stretching/shrinking cylinder in the presence of suction.

Heat transfer characteristics of the stretching sheet problem have been restricted to two boundary conditions of either prescribed temperatures or heat flux on the wall in the published papers. Most recently, the heat transfer problems for a boundary layer flow concerning a convective boundary condition was investigated by Aziz for Blasius flow [35]. Following Aziz, many researchers investigated boundary layer flows with the convective boundary condition [36–40].

All the above-mentioned studies on the flow along a stretching cylinder have been carried out with a prescribed surface temperature (PST) and prescribed surface heat flux (PHF) boundary conditions only. This motivates us to do the present work. We investigated the slip effect on MHD flow along a vertical stretching cylinder with a convective boundary condition. Using a similarity transformation, the governing system of partial differential equations was first transformed into coupled nonlinear ordinary differential equations and then solved through the numerical shooting method.

2. Equations of motion

Let us consider the steady axisymmetric slip flow of incompressible fluid along a vertical stretching cylinder in the presence of a uniform magnetic field. The x-axis is measured along the tube axis and the r-axis is measured in the radial direction. It is assumed that a uniform magnetic field of intensity B_0 acts in the radial direction. The magnetic Reynolds number

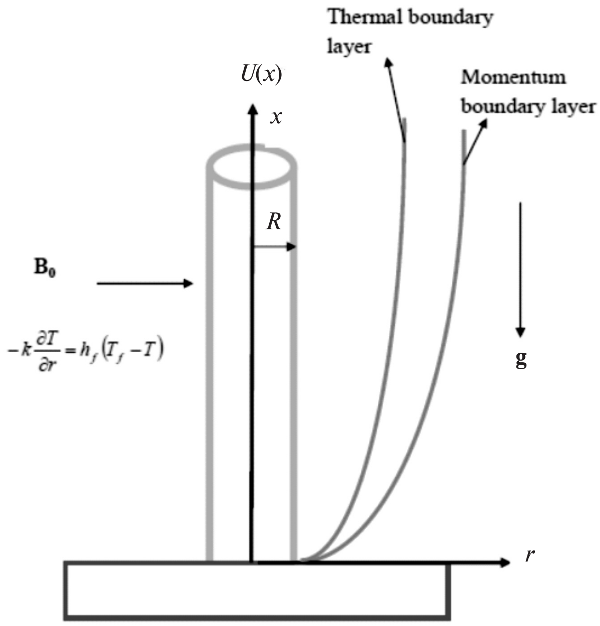

Fig. 1. The coordinate system and the physical model

is assumed to be small so that the induced magnetic field is negligible in comparison with the applied magnetic field. It is also assumed that the left side of the cylinder is heated by convection from the hot fluid at temperature T_f, which provides a heat transfer coefficient h_f, and T is the ambient fluid temperature (see Fig. 1).

The continuity, the momentum and the energy equations governing such type of flow [33] are as follows:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\left(\frac{\partial u}{\partial r}\right)\right) + g\beta(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho}, \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\left(\frac{\partial T}{\partial r}\right)\right), \qquad (3)$$

where u and v are the velocity components, respectively in x and r directions; $\nu = \mu / \rho$ is the kinematic viscosity (μ is the dynamic viscosity of the fluid); ρ is the fluid density; g is the acceleration due to gravity; β is the volumetric coefficient of thermal expansion; σ is the electric conductivity of the medium, B_0 is the uniform magnetic field, α is the thermal diffusivity of the fluid; T is the fluid temperature.

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by the following formulae:

$$u = U(x) + A_1 v\frac{\partial u}{\partial r},\ v = 0,$$
$$-k\frac{\partial T}{\partial r} = h_f(T_f - T) \tag{4}$$

at $r = R$;

$$u \to 0,\ T \to T_\infty \text{ as } r \to \infty. \tag{5}$$

Here $U(x) = U_0x/L$ is the stretching velocity (U_0 is the reference velocity, L is the characteristic length), A_1 is the velocity slip, k is the thermal conductivity, T_f is the hot fluid temperature, h_f is the convective heat transfer coefficient, R is the cylinder radius.

2.2. Method of solution

The continuity equation is automatically satisfied by the introduction of a stream function ψ as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial r},\ v = \frac{-1}{r}\frac{\partial\psi}{\partial x}.$$

Introducing the similarity variable as

$$\eta = \frac{r^2 - R^2}{2R}\left(\frac{U}{\nu x}\right)^{0.5},$$

$$\psi = (U\nu x)^{0.5} R\, F(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{\Delta T},\ \Delta T = T_f - T_\infty \tag{6}$$

and upon substitution of (6) in Eqs. (2) – (5) the governing equations and the boundary conditions are reduced to

$$(1 + 2C\eta)F''' + 2CF'' + FF'' - $$
$$- F'^2 - M^2F' + \lambda\theta = 0, \tag{7}$$

$$(1 + 2C\eta)\theta'' + 2C\theta' + \Pr(F\theta' - F'\theta) = 0, \tag{8}$$

$$F = 0,\ F' = 1 + AF'',$$
$$\theta' = -\gamma(1 - \theta) \text{ at } \eta = 0, \tag{9}$$

$$F' \to 0,\ \ \theta \to 0 \text{ as } \eta \to \infty, \tag{10}$$

where the prime denotes differentiation with respect to η; A is the slip parameter, M^2 is the magnetic parameter,

$$A = A_1\left(\frac{U_0\nu}{L}\right)^{0.5};\ \ M^2 = \frac{\sigma B_0^2 L}{\rho U_0};$$

C is the curvature parameter, λ is the mixed convection parameter,

$$C = \left(\frac{\nu L}{U_0R^2}\right)^{0.5};\ \ \lambda = \frac{g\beta\Delta T_0 L^2}{U_0^2};$$

Pr is the Prandtl number, γ is the surface convection parameter,

$$\Pr = \frac{\mu C_p}{k};\ \ \gamma = \frac{h_f}{k}\left(\frac{L\nu}{U_0}\right)^{0.5}.$$

We assume that $\Delta T = \Delta T_0 x$.

Other important characteristics of the present investigation are the skin friction coefficient C_f and the Nusselt number N_u which are defined as follows:

$$C_f = \frac{2\mu\left(\frac{\partial u}{\partial r}\right)_{r=R}}{\rho U_w^2} = 2F''(0)(\mathrm{Re}_x)^{-0.5};$$

$$N_u = -\frac{x\left(\frac{\partial T}{\partial r}\right)_{r=R}}{T_w - T_\infty} = -(\mathrm{Re}_x)^{0.5}\theta'(0).$$

Here Re_x is the local Reynolds number,

$$\mathrm{Re}_x = \frac{Ux}{\nu}.$$

2.4. Analytical solutions for a special case

It is significant that if $C = 0$ (i.e. $R \to \infty$), the problem under consideration (with $\lambda = M = A = 0$ and $\gamma \to \infty$) is reduced to the boundary layer flow along a stretching at the plate. The closed form of the solution of Eq. (7), with the boundary conditions given in Eqs. (9) and (10), in the absence of λ and C is obtained as

$$f(\eta) = \left(\frac{1 - e^{\beta\eta}}{A\beta^2 + \beta}\right), \tag{11}$$

where

$$\beta = -(3A)^{-1} - \left(\frac{2^{1/3}\beta_1}{3A(\beta_2 + \sqrt{4\beta_1^3 + \beta_2^2})^{1/3}}\right) +$$

$$+\left(\frac{(\beta_2+\sqrt{4\beta_1^3+\beta_2^2})^{1/3}}{3A2^{1/3}}\right)$$

with

$$\beta_1=-(1+3M^2A^2),$$

$$\beta_2=A^2(27+18M^2)-2.$$

The analytical solution of Eq. (8) with the boundary conditions in Eqs. (9) and (10) is obtained by the power series method in terms of Kummer's hypergeometric function [41] as

$$\theta(\eta)=c_1e^{-a_0\beta\eta}M\left(a_0-1,a_0+1,\frac{-\Pr X}{\beta^2}e^{-\beta\eta}\right). \quad (12)$$

The heat transfer rate at the surface is given by

$$\theta'(0)=c_1\left(-a_0\beta M\left(a_0-1,a_0+1,\frac{-\Pr X}{\beta^2}\right)+\right.$$
$$\left.+\left(\frac{a_0-1}{a_0+1}\right)\left(\frac{\Pr X}{\alpha}\right)M\left(a_0,2+a_0,\frac{-\Pr X}{\beta^2}\right)\right), \quad (13)$$

where

$$c_1=-\gamma\left(\frac{\Pr X}{\beta}\left[a_0\left(\frac{-\beta^2}{\Pr X}\right)\times\right.\right.$$
$$\times M\left(a_0-1,a_0+1,\frac{-\Pr X}{\beta^2}\right)+\left(\frac{a_0-1}{a_0+1}\right)\times$$
$$\left.\times M\left(a_0,2+a_0,\frac{-\Pr X}{\beta^2}\right)\right]-$$
$$\left.-\gamma M\left(a_0-1,a_0+1,\frac{-\Pr X}{\beta^2}\right)\right)^{-1}$$

$$a_0=\frac{\Pr X}{\beta^2},\quad X=(A\beta+1)^{-1}.$$

2.5. Numerical method for solution

The nonlinear coupled differential Eqs. (7) and (8) along with the boundary conditions (9) and (10) form a two-point boundary value problem. This problem is solved using the shooting technique together with the fourth-order Runge-Kutta integration scheme by converting it into an initial value problem. In terms of this method we have to choose a suitable finite value of $\eta\to\infty$, for instance η_∞. We set the following first-order system:

$$y_1'=y_2,$$
$$y_2'=y_3,$$
$$y_3'=(1+C\eta)^{-1}(y_2^2+M^2y_2-$$
$$-2Cy_3-y_1y_3-\lambda y_4), \quad (14)$$
$$y_4'=y_5,$$
$$y_3'=-(1+C\eta)^{-1}[2Cy_5+\Pr(y_1y_5-y_2y_4)]$$

with the following boundary conditions:

$$y_1(0)=0,\quad y_2(0)=1+Ay_3(0),$$
$$y_4(0)=1+\left(\frac{y_5(0)}{\gamma}\right). \quad (15)$$

To solve Eq. (14) with conditions (15) as an initial value problem, the values for $y_3(0)$, i.e., $F''(0)$, and $y_5(0)$, i.e. $\theta'(0)$, are required, but such values are not given. In order to obtain the solution, we chose the initial guess values for $F''(0)$ and $\theta'(0)$ and applied the fourth-order Runge-Kutta integration scheme. Then we compared the calculated values of $F'(\eta)$ and $\theta(\eta)$ at η_∞ with the given boundary conditions and adjusted the values $F''(0)$ and $\theta'(0)$ using the shooting technique to achieve the better approximation for the solution. The process was repeated until we obtained the correct results up to the desired accuracy of 10^{-9} level, which fulfilled the convergence criterion.

3. Results and discussion

For the verification of the present numerical results, we have compared $F''(0)$ values with those obtained by Mukhopadhyay [11] in the absence of a slip and mixed convection parameters which are presented in Table 1. The comparison results are found to be good.

In order to analyze the obtained results, the numerical computations were carried out for various values of the curvature parameter C, the mixed convection parameter λ, the magnetic parameter M, the Prandtl number Pr and the surface convection parameter γ. The numerical values are plotted in Figs. 2–6 for illustration of the results. The values of the skin friction coefficient and the Nusselt number have been calculated and are presented in Table 2.

The effects of C and A on the dimensionless horizontal velocity profile are presented

Table 1

Comparison between the results obtained for the local skin friction coefficient $F''(0)$

Magnetic parameter	Mukhopadhyay [11] (without slip parameters)	Present results (without the slip and λ)
	Without curvature	
0.5	−1.11803400	−1.11803399
1.0	−1.41421350	−1.41421356
1.5	−1.80277564	−1.80277564

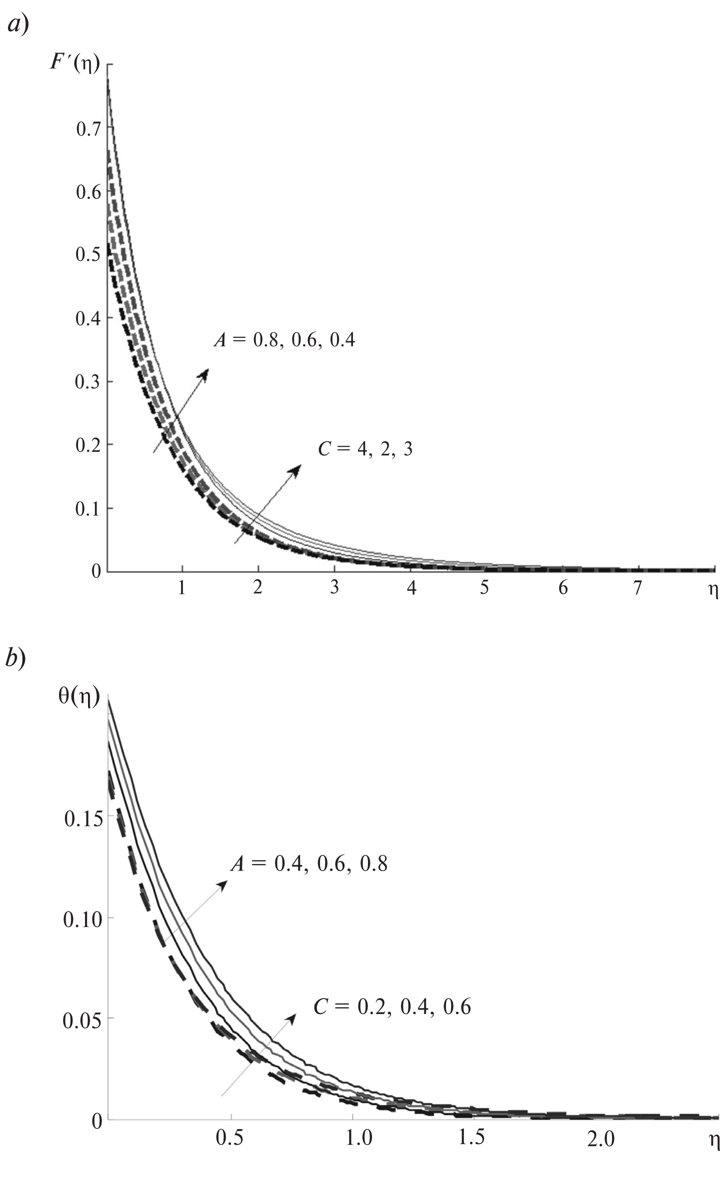

Fig. 2. Effects of slip and curvature parameters on the velocity (*a*) and the temperature (*b*) profiles with $M = 1$, $\lambda = 1$, $\Pr = 8$, $\gamma = 0.6$;
a − solid lines correspond to *C* variation with $A = 0.2$ (variation I), dashed ones to *A* variation with $C = 0.1$ (variation II); *b* − solid lines correspond to variation II, dashed ones to variation I

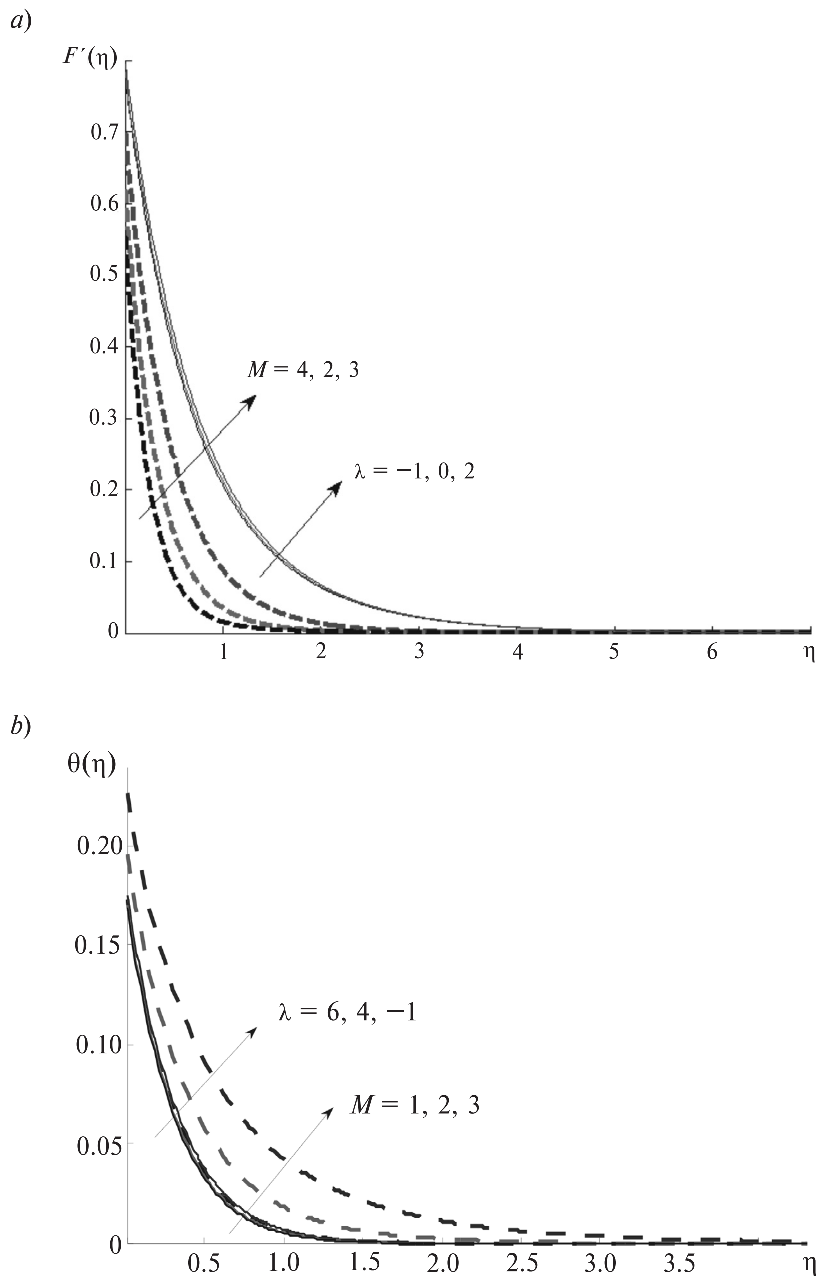

Fig. 3. Effects of magnetic and mixed convection parameters on the velocity (*a*) and the temperature (*b*) profiles with $C = 0.1$, $A = 0.2$, $\mathrm{Pr} = 8$, $\gamma = 0.6$.
Solid lines correspond to λ variation with $M = 1$, dashed ones do to M variation with $\lambda = 1$

in Fig. 2, *a*. It is clear that the curvature parameter decreases the velocity profile near the wall and increases it far away from the wall. When a slip occurs, the flow velocity near the stretching wall is no longer equal to the stretching velocity of the wall. With an increase in A such a slip velocity increases and consequently the fluid velocity decreases because under the slip condition, the pulling of the stretching wall can only be partly transmitted to the fluid. Thus, the increasing values of the slip parameter decelerate the dimensionless velocity profile.

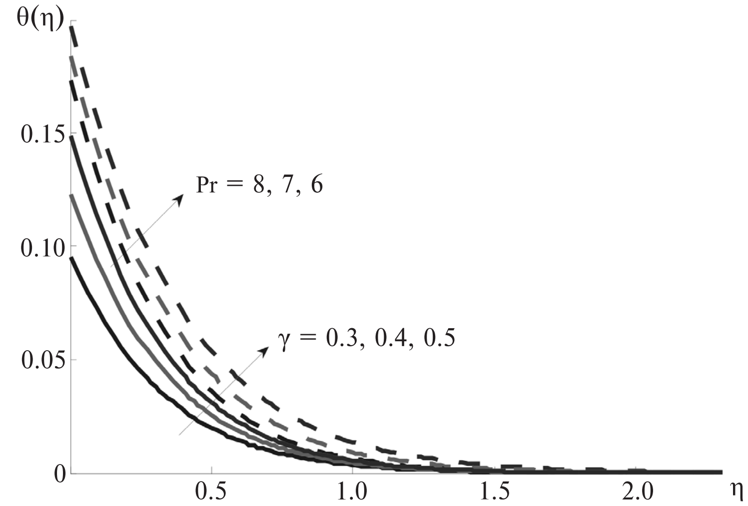

Fig. 4. Effects of the Prandtl number and the surface convection parameter on the temperature profile with $C = 0.1$, $A = 0.2$, $M = \lambda = 1$;
γ variation with Pr = 8 (solid lines) and Pr variation with $\gamma = 0.6$ (dashed ones) are presented

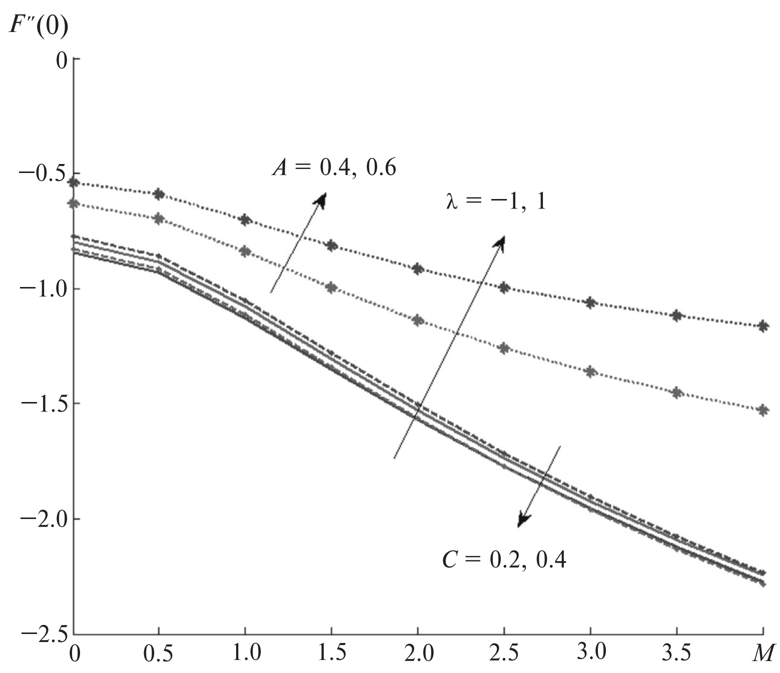

Fig. 5. Effects of curvature, slip, mixed convection and magnetic parameters on $F''(0)$.
C variation with $M = 1$, $\lambda = 1$, Pr = 8, $A = 0.2$ and $\gamma = 0.6$ (solid lines); λ variation with $M = 1$, $C = 0.1$, Pr = 8, $A = 0.2$ and $\gamma = 0.6$ (dashed ones); A variation with $M = 1$, $C = 0.1$, Pr = 8, $\lambda = 1$, and $\gamma = 0.6$ (dotted lines with stars) are presented

Fig. 3, *a* depicts the effects of M and λ on the horizontal velocity profile. It can be seen that increasing values of the magnetic parameter reduce the horizontal velocity profile. The figure clearly shows that the transverse magnetic field opposes the transport phenomena. This is due to the fact that M variation leads to the variation of the Lorentz force induced by the magnetic field, and this force produces more resistance to transport phenomena. With an increase in λ, the horizontal velocity is found to increase the buoyancy-induced flow ($\lambda > 0$) but decreases the buoyancy-opposed one ($\lambda < 0$). For $\lambda > 0$, there is a favorable pressure gradient caused by the buoyancy forces, which results in the flow being accelerated.

The effects of C and A on the dimensionless temperature profile are shown in Fig. 2, *b*. It can be seen that increasing the values of the curvature parameter enhances the thermal boundary-layer thickness. The temperature profile increases with an increase in the slip parameter. This is because an increase in the slip parameter leads to the thickening of the thermal boundary layer.

Fig. 3, *b* demonstrates the effects of λ and M on the temperature profile. With increasing λ, the temperature is found to decrease in the

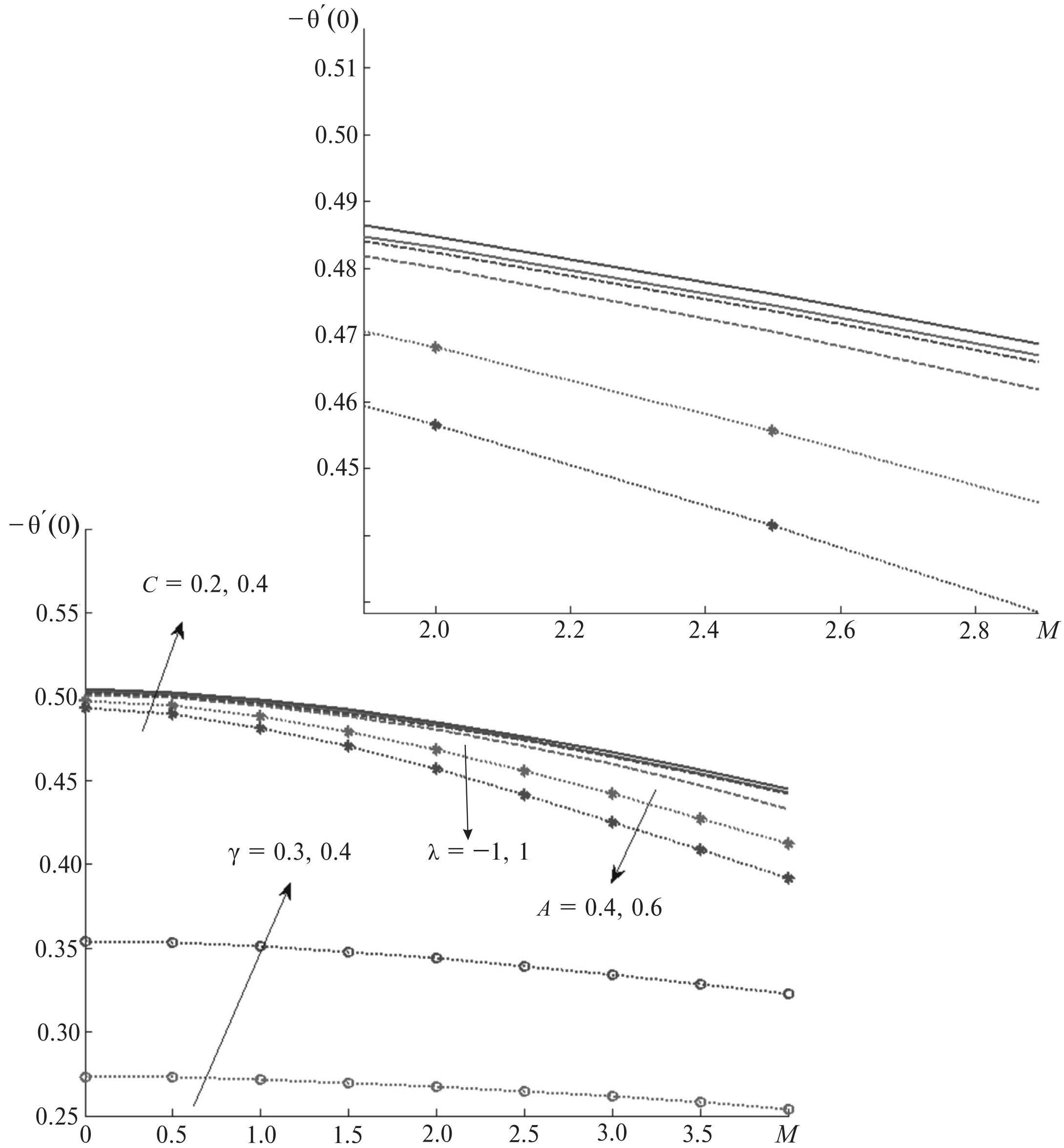

Fig. 6. Effects of curvature, slip, mixed convection, surface convection and magnetic parameters on $\theta'(0)$. Here we present: C variation with $M = 1$, $\lambda = 1$, Pr = 8, $A = 0.2$ and $\gamma = 0.6$ (solid lines); λ variation with $M = 1$, $C = 0.1$, Pr = 8, $A = 0.2$ and $\gamma = 0.6$ (dashed ones); A variation with $M = 1$, $C = 0.1$, Pr = 8, $\lambda = 1$, and $\gamma = 0.6$ (dotted lines with stars); γ variation with $M = 1$, $C = 0.1$, Pr = 8, $\lambda = 1$, $A = 0.2$ (dotted ones with circles)

buoyancy-induced flow and to increase with λ in the buoyancy-opposed one. The thermal boundary-layer thickness decreases with an increase in the values of the mixed convection parameter. The temperature profile enhances with an increase in the magnetic parameter. Thus, the presence of the magnetic field leads to increasing the thermal boundary-layer thickness.

The effects of Pr and γ on the dimensionless temperature profile are elucidated in Fig. 4. It is clear that the increasing values of Pr reduce the temperature profile. The fluids with lower Prandtl numbers have higher thermal conductivities (and thicker thermal boundary-layer structures) so that heat can diffuse from the sheet faster than for fluids with higher Pr numbers (that have thinner boundary-layers). Hence the Prandtl number can be used to increase the rate of cooling in conductive fluids. It is also seen that the temperature profile enhances with the surface convection parameter. The γ parameter is directly proportional to the heat transfer coefficient associated with the hot fluid h_f. The thermal resistance on the hot fluid side is inversely proportional to h_f. Thus, as γ increases, the hot fluid side convection resistance decreases and consequently, the temperature profile increases.

Physically, the negative sign of $F''(0)$ implies that the surface exerts a dragging force on the fluid and the positive sign implies the opposite effect. This is consistent with the present case, as a stretching cylinder which induces the flow is considered here. Fig. 5 shows the variation of the skin friction coefficient. It is clear that this coefficient $F''(0)$ grows with increasing λ and A, and decreases with increasing C and M. The variation of the Nusselt number is shown in Fig. 6. It is observed that the Nusselt number grows in value with increasing values of C, λ and γ, and decreases with M and A.

Table 2

Various values of governing parameters and $F''(0)$, $\theta'(0)$ values for them

Governing parameter	Value		
	Governing parameter	$-F''(0)$	$-\theta'(0)$
C	0.1	1.06754708	0.18479261
	0.2	1.09310236	0.18493968
	0.3	1.11775841	0.18508489
M	1	1.06754708	0.18479261
	2	1.52007910	0.18220507
	3	1.91790594	0.17847387
λ	1	1.06754708	0.18479261
	2	1.05348361	0.18484647
	3	1.03963697	0.18489862
A	0.2	1.06754708	0.18479261
	0.3	0.94852567	0.18407353
	0.4	0.85471337	0.18341212
Pr	6	1.06754708	0.18479261
	7	1.06950873	0.18595638
	8	1.07099257	0.18689031
γ	0.1	1.07438124	0.09604074
	0.2	1.06754708	0.18479261
	0.3	1.06125922	0.26708712

Note: While studying the effect of individual parameters the following values were assumed: $C = 0.1$, $M = 1$, $\lambda = 1$, $A = 0.2$, $\mathrm{Pr} = 6$, $\gamma = 0.2$.

4. Conclusion

The steady axisymmetric laminar boundary-layer slip flow of viscous incompressible fluid and heat transfer towards a vertical stretching cylinder in the presence of the uniform magnetic field has been investigated. Using a similarity transformation, the governing system of partial differential equations was first transformed into coupled nonlinear ordinary differential equations and then was solved with the help of the numerical shooting method. The parameters involved in this study significantly affect the flow and heat transfer. The following conclusions can be drawn as a result of the computations:

(*i*) The thickness of the momentum boundary layer decreases in the presence of the velocity slip and the magnetic field and increases with the mixed convection and curvature parameters;

(*ii*) The thermal boundary-layer thickness increases with an increase in the velocity slip, the magnetic field, the surface convection parameter and the curvature parameter, and decreases with the Prandtl number;

(*iii*) The skin friction coefficient enhances with the increasing values of mixed convection slip and surface convection parameters and decreases with the curvature parameter, the magnetic parameter and the Prandtl number. The Nusselt number increases with the curvature parameter, the mixed convection parameter, the Prandtl number and the surface convection parameter, and decreases with magnetic and slip parameters.

We expect that the physics of flow over a stretching cylinder can be utilized as a basis for many engineering and scientific applications especially in petroleum engineering with the help of the present model. The results pertaining to the present study may be useful for investigating different models. The findings of the present problem are also of great interest in different areas of science and technology, where the surface layers are being stretched.

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Ганеш Н.В., Ганга Б., Хаким А.К.А., Саранья С., Калайванан Р. ГИДРОМАГНИТНЫЙ ОСЕСИММЕТРИЧНЫЙ ОБТЕКАЮЩИЙ ПОТОК ВДОЛЬ ВЕРТИКАЛЬНОГО РАСТЯНУТОГО ЦИЛИНДРА С КОНВЕКТИВНЫМ ГРАНИЧНЫМ УСЛОВИЕМ.

Исследован стационарный осесимметричный ламинарный обтекающий поток вязкой несжимаемой жидкости в пограничном слое, а также его теплопередача по направлению к вертикальному растянутому цилиндру в однородном магнитном поле. Сделано предположение, что левая поверхность цилиндра нагревается горячим конвективным потоком. В качестве первого шага мы преобразовали определяющую систему дифференциальных уравнений с частными производными в систему парных нелинейных обыкновенных дифференциальных уравнений, используя преобразование подобия. Полученная сложная нелинейная краевая задача была решена численно методом Рунге-Кутты 4-го порядка с помощью итерационной процедуры пристрелки. Для специального случая нами представлены аналитические решения. Влияние различных физических параметров на профили скорости и температуры обсуждается с привлечением графического материала. Значения коэффициента поверхностного трения и числа Нуссельта сведены в таблицы и проанализированы. Установлено, что толщина термического граничного слоя увеличивается с ростом скоростного скольжения, величины магнитного поля, параметров поверхностной конвекции и кривизны и уменьшается с ростом числа Прандтля.

ОСЕСИММЕТРИЧНЫЙ ПОТОК, МАГНИТОГИДРОДИНАМИКА, ОБТЕКАЮЩИЙ ПОТОК, РАСТЯНУТЫЙ ЦИЛИНДР, КОНВЕКТИВНАЯ ГРАНИЦА.

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